Observer for DC voltages in a cascaded H-bridge multilevel STATCOM

J. de León Morales, M.F. Escalante and M.T. Mata-Jiménez

Abstract: Using an instantaneous model of a cascade H-bridge multilevel-based STATCOM, an observer for estimating the DC capacitor voltages of the multilevel inverter is presented. This design can be used to eliminate the voltage sensors in each H-bridge of this device. Simulation results are presented in order to validate the performance of the proposed observer applied to a cascade H-bridge multilevel-based STATCOM.

1 Introduction

Cascaded H-bridge multilevel converters represent an important class of a new breed of power converters offering several advantages in handling electrical power over the classical two-level topologies [1–5]. In fact, the H-bridge multilevel converter is composed of several two-level H-bridge inverters connected in series. Each H-bridge is fed by a separate and isolated DC power supply. This configuration allows to handle high voltages using low-voltage semiconductor devices and thus increasing the voltage and power rating of the converter. Higher-level inverters can be easily implemented by adding additional H-bridges.

Cascaded H-bridge multilevel structures offer several advantages in electrical network compensation [6–9]. Their modular and simple topology is very attractive and the needed DC voltage sources can be assured by simple capacitors. When capacitors are used to assure the DC side of each inverter, the voltage at their terminals must be known and controlled. Thus, regulation of these capacitor voltages requires information about such voltages. For that there are two possibilities either measuring or estimating such voltages.

Regarding the first possibility, it presents several disadvantages: besides the known difficulties when measuring high-voltage levels, if the number of cascaded H-bridges increases, then it will be necessary to use multiple voltage sensors to measure the capacitor voltages, which reduces the reliability. On the other hand, the second alternative avoids the use of physical sensors which are replaced by software sensors, called observers. This is a more attractive solution because it takes advantage of the mathematical model of the system. They are less expensive and more reliable because they are implemented by software in a digital computer. Nowadays, the computation capabilities of digital signal processors (DSPs) enables the implementation of such sophisticated and complex algorithms with high degree of performance.

On the other hand, from a control theory point of view, an observer is a computational algorithm to reconstruct the state information of the system from the inputs, the mathematical model and measurable outputs of the actual system. Recently, several works related to observer design for nonlinear systems have been published (see, for instance [10–13]).

This paper proposes a nonlinear observer in order to estimate the capacitor voltages of a cascaded H-bridge multilevel inverter when used as reactive power and harmonics compensator, avoiding the use of voltage sensors. This approach uses an instantaneous model of the system in order to reconstruct the missing state information which is not available from sensors. The model will be represented as a set of n interconnected subsystems for which it will be possible to design an observer.

This paper is organised as follows. In Section 2, a model of the cascaded H-bridge multilevel inverter is presented. The observer design for the cascaded H-bridge multilevel inverter is given in Section 3. In Section 4, simulation results are given in order to illustrate the performance of the proposed observer to estimate the H-bridge capacitor voltages. Finally, some conclusions are given.

2 Modelling of cascaded H-bridge multilevel inverters

Cascaded H-bridge multilevel inverters are based on serial connection of single-phase inverters bridges. A single phase leg is shown in Fig. 1. As shown, each inverter bridge is supplied by a separate and isolated DC power supply. When it is used to compensate reactive power and harmonics in electrical networks, a simple charged capacitor can provide the needed voltage in the DC side of the i th inverters.

The output voltage of a cascaded H-bridge inverter leg is obtained by adding the single H-bridge output voltages as follows

\[ v_i(t) = \sum_{j=1}^{n} v_{ij}(t) \]  

(1)

If the DC side of each H-bridge has a value of \( V_{dc} \) volts, then the output voltage of each individual H-bridge can take three different values: \(-V_{dc}, 0\) or \( +V_{dc}\), as a function of the switching states, and thus \( v_0 \in \{-V_{dc}, 0, +V_{dc}\}\). Taking into account that each H-bridge inverter leg is
composed of a series connected pair of switches, \((T_{1j}, T_{2j})\) and \((T_{3j}, T_{4j})\), those switches must be commanded in a complementary way. Following this rule, it is possible to define a switching function, \(S_j\), to describe the output voltage of each \(j\)th H-bridge. This switching function can take three different values: \(-1, 0, 1\), and is directly related to the switching states. In Table 1, the switching function, \(S_j\), as a function of the switching states is shown. It is worth noting that the given switching function can be used to describe the H-bridge output voltage for both bipolar and unipolar modulation, because it fully describes the switching states generated by both types of modulation.

Introducing the switching function in (1), then it follows that

\[
v_o(t) = \sum_{j=1}^{n} V_{dcj} S_j
\]  

(2)

If a capacitor is used at the DC side of each H-bridge, as shown in Fig. 2, then the capacitor voltage for the \(j\)th bridge \(v_{cj}\) can be described by

\[
\frac{d}{dt} v_{cj} = \frac{1}{C} i(t) S_j
\]  

(3)

where \(C\) is the capacitance. Thus, the inverter phase voltage \(v_o(t)\) can be expressed as

\[
v_o(t) = \sum_{j=1}^{n} v_{cj}(t) S_j
\]  

(4)

The equivalent circuit for one phase when the multilevel inverter is connected to the electrical network is shown in Fig. 3. Furthermore, the phase current \(i(t)\), as represented in Fig. 3 is given by

\[
\frac{d}{dt} i(t) = -\frac{1}{L} \left[ v_o(t) - v_{net}(t) \right] - \frac{r}{L} i(t)
\]  

(5)

where \(v_{net}(t)\) is the network phase voltage and \(L\) and \(r\) are the equivalent inductance and resistance of the linker transformer or reactor, respectively.

Now using (2), (4) and (5), the dynamical model describing the behaviour of the \(j\) H-bridges connected per phase is given by

\[
\begin{align*}
\frac{d}{dt} v_{cj}(t) &= \frac{1}{C} i(t) S_j, \\
\frac{d}{dt} v_{o}(t) &= \frac{1}{C} i(t) \sum_{j=1}^{n} S_j \\
\frac{d}{dt} v_{o}(t) &= \frac{1}{C} i(t) S_j \\
... &... \\
\frac{d}{dt} v_{o}(t) &= \frac{1}{C} i(t) S_j \\
y &= i(t)
\end{align*}
\]

(6)

The resulting system is a \((n+1)\)-dimensional nonlinear system, where the output is given by \(y = i(t)\), and \(S_j\) for \(j = 1, \ldots, n\) are the \(n\) inputs of the system. An observer is proposed to estimate the capacitor voltages of the cascaded H-bridge multilevel inverter without using physical sensors. Furthermore, assuming that the current \(i(t)\) is the only measurable variable of the system (6), from the observability rank condition, it follows that \(\text{dim}(d\mathcal{O}) = 2\), where \(d\mathcal{O}\) is the space of differential elements of the observation space \(\mathcal{O}\) (see Definiton 2 in Appendix A for more details). It is clear that \(d\mathcal{O}\) is not of full rank, that is, the system is not observable. To overcome this difficulty, a convenient representation of the model is proposed and a methodology taking advantage of this representation is developed in the next section.

### 3 Observer design

In this section, an alternative representation of model (6) will be considered, that is, the original model will be split into a suitable set of \(n\) subsystems for which it is possible

---

**Table 1: Switching function**

<table>
<thead>
<tr>
<th>(T_1)</th>
<th>(T_3)</th>
<th>(S_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFF</td>
<td>OFF</td>
<td>0</td>
</tr>
<tr>
<td>OFF</td>
<td>ON</td>
<td>-1</td>
</tr>
<tr>
<td>ON</td>
<td>OFF</td>
<td>1</td>
</tr>
<tr>
<td>ON</td>
<td>ON</td>
<td>0</td>
</tr>
</tbody>
</table>
to design an observer for estimating the capacitor voltages \( v_{cj}, j = 1, \ldots, n \). Equation (6) can be split into \( n \) subsystems of the form

\[
\begin{align*}
\frac{d}{dt}(t) &= -r_c \frac{v_{cj}}{l_c} - \frac{1}{l_c} S_j v_{cj} + \frac{1}{l_c} (S_2 v_{c1} + \cdots + S_n v_{cn} - v_{net}), \\
\frac{d}{dt} v_{cj} &= \frac{1}{C_j} S_j i(t), \\
y &= i(t),
\end{align*}
\]

The new set of subsystems represented will be used to design an observer for the \( n \) capacitor voltages. Each subsystem can be written in a state affine form as

\[
\begin{align*}
\mathbf{x}_j &= \mathbf{A}(S_j) \mathbf{x}_j + \mathbf{G}_j \mathbf{v}_j, \\
y &= C_j \mathbf{x}_j, \\
\mathbf{v}_j &= (v_{cj}, \ldots, v_{cn}, \ldots, v_{cj}, v_{cn})^T,
\end{align*}
\]

where \( \mathbf{x}_j = (i, v_{cj})^T \) represents the state of the \( j \)th subsystem, and \( S_j, j = 1, \ldots, n \), are the instantaneous inputs applied to the system and \( y = i(t) \) is the measurable output, with

\[
\mathbf{A}(S_j) = \begin{pmatrix}
-a_j & -1 / l_c \\
1 / C_j & 0
\end{pmatrix}
\]

\[
\mathbf{G}_j \mathbf{v}_j = B \varphi(S_j, \mathbf{x}_j, v_{net})
\]

\[
B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad C_j = (1 \ 0), \quad j = 1, \ldots, n
\]

and

\[
\varphi(S_j, \mathbf{x}_j, v_{net}) = -\frac{1}{l_c} v_{net} - \sum_{m=1}^{n} v_{cm} S_m
\]

where \( S_j = (S_1, \ldots, S_{j-1}, S_{j+1}, \ldots, S_n) \) and \( \mathbf{v}_j = (v_{c1}, \ldots, v_{cj}, v_{cn}, \ldots, v_{c1}, v_{cn})^T \).

The function \( \varphi(S_j, \mathbf{x}_j, v_{net}) \) is the interconnection term depending on inputs and states of each subsystem. Notice that the output is the current \( i(t) \) and is the same for each subsystem as shown in Fig. 4.

The new set of subsystems represented will be used to design an observer for the \( n \) capacitor voltages.

Fig. 4 Observer block diagram

Then, the following system

\[
\begin{align*}
\mathbf{z}_j &= \mathbf{A}(S) \mathbf{z}_j + \mathbf{G}_j (S_j, \mathbf{z}_j) - P_j^{-1} C_j^T (y - \hat{y}), \\
P_j &= -\Theta P_j - A^T(S) P_j - P_j A(S) + C_j^T C_j
\end{align*}
\]

is an observer for the system (7) for \( j = 1, \ldots, n \) where \( P_j^{-1} C_j^T \) is the gain of the observer which depends on the solution of the Riccati equation (8) for each subsystem and \( \mathbf{z}_j = (\hat{v}_{c1}, \ldots, \hat{v}_{cn}, \ldots, \hat{v}_{c1}, \hat{v}_{cn}) \). Furthermore, the estimation error \( \epsilon_j = z_j - x_j \) converges to zero as \( t \) tends to infinity. The parameter \( \Theta > 0 \) is called the forgetting factor and it determines the convergence rate of the observer (see Appendix A for more details).

It is worth noting that this observer is the deterministic version of the Kalman filter for state affine systems.

It is clear that the observability of the system depends on the applied input as stated in Definition 3. Then the convergence of this observer can be proved assuming that the inputs \( S_j \) are regularly persistent, that is, it is a class of admissible inputs that allows to observe the system (for more details, see [5, 6]). This guarantees the observer works and the observer gain is well defined, that is, the matrix \( P_j \) is nonsingular.

Now, a further result based on regular persistence is introduced.

Lemma 1: Assume that the input \( S_j \) is regularly persistent for system (7) and consider the following Lyapunov differential equation

\[
P_j = -\Theta P_j - A^T(S) P_j - P_j A(S) + C_j^T C_j
\]

with \( P_j > 0 \). Then, \( \exists \theta_0 > 0 \) such that for any symmetric positive definite matrix \( P_j(0), \exists \theta_0 \geq \theta_0, \exists \alpha_j, \beta > 0, t_0 > 0, \forall t > t_0 \)

\[
\alpha_j I < P_j(t) \leq \beta I
\]

where \( I \) is the identity matrix.

The proof of this lemma follows the same steps of Theorem 3.2 in [14].
Now, consider that the system $\sum_{NL}$ can be represented as a set of the interconnected subsystems as follows

$$\begin{align*}
\sum_j: \begin{cases}
    \dot{x}_1 = A(S_j)x_1 + IF_1(S_j, \hat{x}_1), \\
    \dot{x}_2 = A(S_j)x_2 + IF_2(S_j, \hat{x}_2), \\
    \vdots \\
    \dot{x}_n = A(S_j)x_n + IF_n(S_j, \hat{x}_n)
\end{cases}
\end{align*}$$ (10)

Notice that the output is the current $i(t)$ and is the same for each subsystem.

The main idea of this paper is to construct an observer for the whole system $\sum_j$, from the separate design of each subsystem $\sum_j$.

In general, if each $O_j$ is an exponential observer for $\sum_j$, then following interconnected system $O$

$$\begin{align*}
O: \begin{cases}
    \dot{Z}_1 = A(S_j)Z_1 + \Gamma_1(S_j, \hat{Z}_1) - P_1^{-1}C_1^T(y - \hat{y}), \\
    \dot{Z}_2 = A(S_j)Z_2 + \Gamma_2(S_j, \hat{Z}_2) - P_2^{-1}C_2^T(y - \hat{y}), \\
    \vdots \\
    \dot{Z}_n = A(S_j)Z_n + \Gamma_n(S_j, \hat{Z}_n) - P_n^{-1}C_n^T(y - \hat{y}), \\
    \dot{P}_1 = -\theta_1P_1 - A^T(S_j)P_1 - P_1A(S_j) + C_1^TC_1, \\
    \dot{P}_2 = -\theta_2P_2 - A^T(S_j)P_2 - P_2A(S_j) + C_2^TC_2, \\
    \vdots \\
    \dot{P}_n = -\theta_nP_n - A^T(S_j)P_n - P_nA(S_j) + C_n^TC_n
\end{cases}
\end{align*}$$ (11)

is an observer for the interconnected system $\sum_j$.

Remark: The proposed observer $O$ works for inputs satisfying the regularly persistent condition, which is equivalent to each subsystem $\sum_j$ being observable, and hence, observer $O_j$ works at the same time, whereas the other subsystems become observable when their corresponding input satisfies the regularly persistent condition.

Now, we will give the sufficient conditions which ensure the convergence of the interconnected observer $O$. For that, we introduce the following assumptions.

**Assumption 1:** Assume that the input $S_j$ is a regularly persistent input for subsystem $\sum_j$, and admits an exponential observer $O_j$ for $j = 1, \ldots, n$. In this case, an observer of the form (11) can be designed and the estimation error will be bounded.

**Assumption 2:** The term $\Gamma_j(S_j, \hat{x}_j)$ does not destroy the observability property of subsystem $\sum_j$ under the action of the regularly persistent input $S_j$. Moreover, $\Gamma_j(S_j, \hat{x}_j)$ is Lipschitz with respect to $\hat{x}_j$ and uniformly with respect to $S_j$ for $j = 1, \ldots, n$.

The following result can be established.

**Lemma 2:** Considering the interconnected system $\sum_j$ given by (10) and if Assumptions 1 and 2 are satisfied. Then, the system $O$ (11) is an observer for system $\sum_j$ (10).

**Proof:** The dynamics of the estimation error, $e_j = Z_j - \hat{X}_j$, is given by

$$\dot{e}_j = (A(S_j) - P_j^{-1}C_j^TC_j)e_j + \Delta \Gamma_j(S_j, \hat{x}_j, \hat{Z}_j)$$

where

$$\Delta \Gamma_j(S_j, \hat{x}_j, \hat{Z}_j) = \Gamma_j(S_j, \hat{Z}_j) - \Gamma_j(S_j, \hat{x}_j),$$

for $j = 1, \ldots, n$.

Next, from Lemma 1 and Assumption 1 (see Appendix A), let be $V = \sum_{j=1}^n V_j$ a Lyapunov function for the interconnected system $\sum_j$, where $V_j(e_j) = e_j^TP_j e_j$ is a Lyapunov function for each subsystem $\sum_j$. Taking the time derivative of $V_j(e_j)$, it follows that

$$\begin{align*}
\dot{V}_j(e_j) &\leq -\theta V_j(e_j) + e_j^TP_j\Delta \Gamma_j(S_j, \hat{x}_j, \hat{Z}_j) \\
&\quad + e_j^TP_j\Delta \Gamma_j(S_j, \hat{x}_j, \hat{Z}_j) \\
&\quad \pm \Delta \Gamma_j(S_j, \hat{x}_j, \hat{Z}_j)^TP_j\Delta \Gamma_j(S_j, \hat{x}_j, \hat{Z}_j)
\end{align*}$$

where $\Delta \Gamma_j(S_j, \hat{x}_j, \hat{Z}_j) = B\varphi_j(e)$ and $\varphi_j(e) = -1/\epsilon \sum_{j=1}^n S_j e_{j}/v_{ij}$, where $e_{ij} = v_{ij} - \hat{v}_{ij}$.

Now adding and subtracting the term $\Delta \Gamma_j(S_j, \hat{x}_j, \hat{Z}_j)^TP_j\Delta \Gamma_j(S_j, \hat{x}_j, \hat{Z}_j)$, we have

$$\dot{V}_j(e_j) \leq -\theta V_j(e_j) + 2e_j^TP_j\Delta \Gamma_j(S_j, \hat{x}_j, \hat{Z}_j)$$

**Fig. 5** Block diagram of a cascaded H-bridge multilevel-inverter-based compensator and interconnected observer

**Fig. 6** Real compensator phase current
Fig. 7  Capacitor voltages and their estimation errors

a  Real and estimated capacitor voltages

b  Estimation errors (considering nominal capacitance values: $C_1 = C_2 = C_3 = C_4 = 50 \text{ mF}$)
Next, regrouping the appropriate terms

\[ \dot{V}(e_j) \leq -(\theta_j - 1)\| e_j \|^2_{P_j} - \| e_j \|^2_{P_j} + 2e_j \cdot P_j \Delta \Gamma_j(S_j, \tilde{\chi}_j, \tilde{Z}_j) \]

\[ -\| \Delta \Gamma_j(S_j, \tilde{\chi}_j, \tilde{Z}_j) \|^2_{P_j} + \| \Delta \Gamma_j(S_j, \tilde{\chi}_j, \tilde{Z}_j) \|^2_{P_j} \]

It follows that

\[ \dot{V}(e_j) \leq -(\theta_j - 1)\| e_j \|^2_{P_j} + \| \Delta \Gamma_j(S_j, \tilde{\chi}_j, \tilde{Z}_j) \|^2_{P_j} \]

Now, from Assumption 2, the inequality \( \| \Delta \Gamma_j \|_{P_j} < \sum_{i \in \mathcal{N}_j} \lambda_i \| e_i \|^2_{P_j} \) holds, we get

\[ \dot{V}(e_j) \leq -(\theta_j - 1)\| e_j \|^2_{P_j} + \sum_{i \in \mathcal{N}_j} \lambda_i \| e_i \|^2_{P_j} \] (12)

Then, the time derivative of \( V \) is given by

\[ \dot{V}(e) = \sum_{j=1}^{n} \dot{V}(e_j) \]

Fig. 8  Real and estimated inverter phase current

Fig. 9  Estimation errors for the inverter phase current
exists a positive constant $m$

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| H-bridge output voltage and individual H-bridge output voltage |

and replacing the expression (12) in the above equation, it follows that

$$\dot{V}(e) \leq \sum_{j=1}^{n} \left\{ -(\theta_j - 1)\|e_j\|_p^2 + \sum_{l=1}^{n} \lambda_l \|e_l\|_p^2 \right\}$$

Taking into account that the inputs are regularly persistent, then from Lemma 1, the matrices $P_j$ are bounded.

Using the lemma on equivalence of norms, that is, it exists a positive constant $\mu_l$ such that

$$\|e_l\|_p^2 \leq \mu_l \|e_l\|_p^2 \quad \forall \ l = 1, \ldots, n$$

Then, it follows that

$$\dot{V}(e) \leq \sum_{j=1}^{n} \left\{ -(\theta_j - 1)\|e_j\|_p^2 + \sum_{l=1}^{n} \lambda_l \mu_l \|e_l\|_p^2 \right\}$$

or

$$\dot{V}(e) \leq -\sum_{j=1}^{n} \left\{ (\theta_j - 1) - (n - 1)\lambda_j \mu_j \right\} \|e_j\|_P^2$$

Finally, we have

$$V(e) \leq V(e(t_0))e^{-\gamma(t-t_0)}$$

for $\gamma = \min(\gamma_1, \ldots, \gamma_n)$ where $\gamma_j = (\theta_j - 1) - (n - 1)\lambda_j \mu_j$. Taking $e = \text{col}(e_1, \ldots, e_n)$, it is easy to see that

$$\|e(t)\| \leq K\|e(t_0)\|e^{-\gamma(t-t_0)}$$

This ends the proof.

In the next section, we validate the performance of the observer by simulation. In order to use the proposed observer schema, a system verifying Assumptions 1 and 2 is considered.

4 Simulation results

A detailed simulation was carried out to validate the proposed interconnected observer. A cascaded H-bridge multilevel-based compensator for reactive power and harmonics was simulated. The general block diagram is shown in Fig. 5. Each multilevel inverter leg has four H-bridges per phase (Block 1). A capacitor $C_{\text{nom}} = 50 \text{ mF}$ (nominal value) per bridge was used. The capacitors were pre-charged at 50 V. The compensator was coupled to a 100-V utility network through a 0.5 mH inductor and 0.1 $\Omega$ resistor (Block 2). Simulated load is a combination of linear and nonlinear elements (Block 3). The whole electrical load connected to the electrical network are four linear and two nonlinear three-phase symmetrical loads. Load parameters are as follows: linear loads 1 and 2: $R = 10 \Omega$, $L = 5 \text{ mH}$. Nonlinear load consists of two three-phase diode rectifiers with the following parameters: Nonlinear load 1: AC input impedance: $R = 0.1 \Omega$, $L = 5 \text{ mH}$, DC side: $L = 1 \text{ mH}$, $C = 2000 \mu F$ and $R_{\text{load1}} = 30 \Omega$. Nonlinear load 2: AC input impedance: $R = 0.1 \Omega$, $L = 0.1 \text{ mH}$, DC side: $L = 50 \text{ mH}$, $C = 0.2 \mu F$ and $R_{\text{load2}} = 5 \Omega$.

The switching frequency was fixed at 5 kHz. Simulations were carried out using PSIM from Powersim and MATLAB/SIMULINK from Mathworks, working in co-simulation mode.

Compensation currents are calculated in Block 4 using the instantaneous power concepts [15]. Basically it calculates compensation currents in such a way that reactive power and oscillating power, produced by load current harmonics, will be supplied by the multilevel compensator. Then compensation currents are controlled using proportional-integral (PI) controllers, the PI gains are $k_p = 0.05$, $k_I = 1$ (Block 5) in a synchronous reference frame. PI current controllers then will generate the voltage references to command the multilevel inverter. To generate the control signals for the multilevel inverter H-bridges, a carrier-based multilevel PWM controller (CPWM) shown in Block 6 is used [16].

The commanded voltages, $v_{cm1}$, $v_{cm2}$ and $v_{cm3}$, calculated

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>1</th>
<th>5</th>
<th>7</th>
<th>11</th>
<th>13</th>
<th>17</th>
<th>19</th>
<th>23</th>
<th>25</th>
<th>29</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>l load, (A)</td>
<td>70.97</td>
<td>10.13</td>
<td>6.98</td>
<td>3.7</td>
<td>2.98</td>
<td>2.23</td>
<td>1.81</td>
<td>1.28</td>
<td>1.06</td>
<td>0.67</td>
<td>0.54</td>
</tr>
<tr>
<td>l supply, (A)</td>
<td>70.26</td>
<td>0.41</td>
<td>1.12</td>
<td>0.5</td>
<td>0.73</td>
<td>0.47</td>
<td>0.44</td>
<td>0.6</td>
<td>0.44</td>
<td>0.26</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 2: Peak values of load and supply harmonic currents (A)
Fig. 12  Capacitor voltages and their estimation errors under non-nominal capacitances

a Real and estimated capacitor voltages

b Estimation errors (considering non-nominal capacitance values: $C_1 = 1.2C_{\text{nom}}$, $C_2 = 0.8C_{\text{nom}}$, $C_3 = 1.1C_{\text{nom}}$, $C_4 = 0.8C_{\text{nom}}$, $C_{\text{nom}} = 50$ mF)
by the PI current controllers are fed into the CPWM in order
to generate the H-bridge control signals. Also in Block 6, the
switching function, $S_{\text{c}}$, is generated as defined by Table 1. $S_{\text{c}}$
will be used in the interconnected observer, Block 7, to
estimate the H-bridge capacitor voltages according to (8)
where the observer gain $d_j = 10,000$ for $j = 1, \ldots, n$. The
details of the observer implementation are given in
Appendix A.

During simulation, the loads were connected at different
times to study the observer behaviour. Nonlinear load 1 is
connected at $t = 0.01$ s, then at $t = 0.15$ s nonlinear load 2
and linear load 1 are connected and finally at $t = 3.5$ s
linear loads 2, 3 and 4 are connected.

Fig. 6 shows the real compensator phase current, where
the current changes correspond to the connection of loads.
Besides current changes due to load connections, at
$t = 1.5$ s it is observed a current change. This change corre-
spond to a direct axis current command in order to force an
increase of capacitor voltages (active power being absorbed
from the utility network).

Fig. 7 shows the real and estimated capacitor voltages for
one phase and their corresponding estimation errors. As
shown, the interconnected observer follows the real capaci-
tor voltages accurately. From $t = 1.5$ to 1.75 s, direct axis
current was commanded to absorb active power from the
electrical network and thus increasing the compensator
current and forcing the capacitor voltages to increase.
Then at $t = 3.5$ s a compensator current step was introduced
due to an increase in reactive current being supplied by the
compensator. In any condition, the observer is able to
follow the real capacitor voltages accurately. Once the
observer transient is over, the estimated voltages are
always close to the real values.

Now, some complementary simulation results are pre-
sented. Fig. 8 shows the real and estimated inverter phase
currents. Notice that different time windows are used to dis-
tinguish the waveforms. The estimated currents are always
close to the real values, even in the case of real current
changes (at $t = 1.5$ and 3.5 s), as shown by the estimation
errors shown in Fig. 9.

Fig. 10 shows the phase voltage, phase current and load
current, whereas Fig. 11 shows the compensator line-to-line
voltage and one bridge individual output voltage. As the test
system (multilevel compensator) was intended to compen-
sate for reactive power and harmonics, phase voltage and
phase current are in phase, which means that reactive
power has been compensated. Also, most harmonics has
been removed from the phase current if compared with the
load current; thus, harmonics have also been compensated.

In Table 2, the load and supply harmonic currents are
shown for $t = 2$ to 3 s. Notice that the harmonics have been
reduced. The THD levels at the load current is 19.3% with a
PF of 0.97 and at the supply current the
THD is only 2.6% with a PF of 0.99. At $t = 3.5$ s the reactive
power has been increased at the load, thus the load PF is
reduced to 0.89 but at the supply the PF is 0.99, which
means that the compensator is effectively compensating for
harmonics and reactive power.

Finally, as the model parameters are always a concern, a
simulation with uncertain capacitance values was con-
ducted. The used capacitance values at each H-bridge were
modified from its nominal values, $C_{\text{nom}} = 50$ mF, as
follows: $C_1 = 1.2C_{\text{nom}}$, $C_2 = 0.8C_{\text{nom}}$, $C_3 = 1.1C_{\text{nom}}$
$C_4 = 0.8C_{\text{nom}}$

Fig. 12 shows the real and estimated capacitor voltages and
their corresponding estimation errors when non
nominal capacitance values are used. As shown, the
observer keeps tracking of the real capacitor voltages, but
its convergence time is increased. Also notice that when a
capacitor voltage is changing rapidly, when real power is
being absorbed from the network (from $t = 1.5$ to 1.75),
the estimation error is increased, but it converges to the
real value when the transient at the capacitor voltages is
over. In future, this problem will be addressed by introdu-
cing an adaptive observer in order to deal with large-
parameter uncertainties or unknown parameter values.

5 Conclusions

The control and monitoring of the voltage of the capacitors
in a cascaded H-bridge multilevel converter is essential for
proper operation of the converter when applied as an elec-
trical network compensator that is, STATCOM. Measuring those voltages becomes expensive and impracti-
cal because of the high voltages and power levels handled in
such applications. Thus the advantages of using an observ-
ation technique becomes evident.

In this paper, an interconnected observer was presented to
estimate the capacitor voltages in an H-bridge multilevel-
based compensator. The proposed observer is based on an
instantaneous model of the system taking into account the
switching process through the modulation function. This
observer can reconstruct the capacitor voltages accurately
and replace the needed capacitor voltage sensors. Notice
that this scheme can be used both for control and monitoring
purposes. The obtained results being acceptable, lets con-
clude that this observer is well suited for purposes of
control and monitoring of the capacitor voltages in a cas-
caded H-bridge multilevel-based compensator.

6 Acknowledgments

This research was supported by PROMEP-CONACyT-
CASEP.

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leads to a necessary but not sufficient condition to be able to design an observer.

In fact, some input $u$ may not allow any distinct initial state pairs to be distinguished. The observability depends on the input and, in general, there exist singular inputs which lead to a lack of observability. This property is fundamental in the observer’s design. This will be verified in the present work.

Now, we introduce some definitions related with the inputs applied to the system. Consider a state-affine controlled system of the form

$$
\dot{x} = Ax + Bu \\
y = Cx
$$

where $x \in \mathbb{R}^n$, $v \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ with $A: \mathbb{R}^n \to \mathbb{R}^n$; $B: \mathbb{R}^m \to \mathbb{R}^n$, $C \in \mathbb{R}^{p \times n}$, and where $\mathcal{M}(k, l)$ denotes the space of $k \times l$ matrices with coefficients in $\mathbb{R}$; $k$ (resp. $l$) is the number of rows (resp. columns). From now on, we will assume that $B(v) = 0$ without loss of generality.

**Notation:** Let $\Phi_i(\tau, t)$ denotes the transition matrix of

$$
\frac{d}{d\tau} \Phi_i(\tau, t) = A(v(\tau))\Phi_i(\tau, t) \\
\Phi_i(\tau, \tau) = I
$$

with the classical relation $\Phi_i(t_1, t_2)\Phi_i(t_2, t_3) = \Phi_i(t_1, t_3)$.

We then define:

- **The Observability Grammian:**
  $$
  \Gamma(t, T, v) = \int_t^{t+T} \Phi_i^T(\tau, T)C^T C \Phi_i(\tau, t) d\tau
  $$

- **The Universality index:**
  $$
  \gamma(t, T, v) = \min_{i} (\lambda_i(\Gamma(t, T, v)))
  $$

where $\lambda_i(M)$ stands for the eigenvalues of a given matrix $M$.

The input functions are assumed to be measurable and such that $A(v)$ is bounded on the set of admissible inputs of $\mathbb{R}^n$. We recall below some required results of input functions ensuring the existence of an observer for (15).

**Definition 3 (Regular persistence):** A measurable bounded input $v$ is said to be regularly persistent for the state-affine system (15) if there exists $T > 0$, $\alpha > 0$ and $t_0 > 0$ such that $\gamma(t, T, v) > \alpha$ for every $t \geq t_0$.

In order to design an observer for systems

$$
\sum_{\lambda \in \mathcal{M}} \left\{ \begin{array}{l}
\dot{\bar{x}} = A(\bar{x}), \\
y = C\bar{x}
\end{array} \right.
$$

we consider a deterministic extension of the Kalman–Bucy filter for state-affine system $\sum_{\lambda \in \mathcal{M}}$. This observer is obtained from the minimisation of a performance index $J(\bar{z}_0)$ which depends on the estimated initial condition $\bar{z}_0$, that is,

$$
J(\bar{z}_0) = e^{-(\lambda_0 - \alpha)T_0} \| z_0 - \bar{z}_0 \|_{\mathcal{L}_2}^2 \\
+ \int_0^T e^{-(\lambda_0 - \alpha)\tau} \| y(\tau) \|
\leq C \Phi_i(\tau, t_0)\bar{z}_0^2 d\tau
$$

8 Appendix A: Observability definitions

In order to construct an observer, it is necessary to verify the observability of the system. Let us first recall some definitions of the observability concept.

**Definition 1:** A system

$$
\dot{x}(t) = f(x(t), u(t)), \\
y(t) = h(x(t))
$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$, is said to be observable if it does not involve indistinguishable pairs $x_0, x_0$ of distinct initial states. Two states $x_0$ and $x_0$ are said to be indistinguishable if, for any input function $u(t)$ and for all $t \geq 0$, the corresponding output $y(t, x_0)$ and $y(t, x_0)$ are equal on their common domain of definition. The system is called observable if $x_0 = x_0$ whenever $x_0$ and $x_0$ are indistinguishable.

For constant parameter linear systems, observability is characterised by the well-known Kalman rank condition. The equivalence for nonlinear systems is not straightforward. The observability concept is in fact global: each point is distinguishable from all the others, even if they are far from each other.

On the contrary, a rank condition is local. The equivalence for nonlinear systems can only be partial. Indeed, the rank condition for nonlinear systems is related to a weak observability concept [17]. This concept can easily be tested by a simple algebraic criterion.

**Definition 2:** The observation space can be defined as

$$
\mathcal{O} = (h, L_1 h, \ldots, L_q h)^T
$$

A system is said to satisfy the observability rank condition at $x_0$ if the dimension of $\mathcal{O}(x_0)$ is $n$ (where $\mathcal{O}$ is the space of the differentials of the elements of $\mathcal{O}$). Moreover, if the system satisfies the observability rank condition at $x_0$ then it is locally weakly observable at $x_0$.

This information does not imply that the system is or is not observable in the sense of Definition 1. The observability rank condition does not give a direct procedure for the construction of an observer but represents a first step in the analysis of the observability study of a system.

The inputs also play an essential role in the observability concept for nonlinear systems, which is not the case for linear systems. Indeed, if a linear system is observable, for any input $u(t)$, we can reconstruct the initial state. This is not true in general for nonlinear systems. The fact that a system is observable in the sense of Definition 1

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where $z_0$ is the initial condition of the observer and $\theta > 0$ is called the forgetting factor which determines the convergence rate of the observer and $S_0$ and $Q$ are definite positive symmetric matrices.

In order to minimise $J(z_0^2)$ with respect to $z_0^2$, set $z_0^2$ to the minimum of $J(z_0^2)$, that is, $\partial J(z_0^2)/\partial z_0^2 = 0$ and defining $z(t) = \Phi_e(t, t_0)z_0$, we have

$$P(t) = e^{-\theta(t-t_0)}\Phi_e^T(t, t_0)P_0\Phi_e(t, t_0) + \int_{t_0}^{t} e^{-\theta(t-\tau)}\Phi_e^T(\tau, t)C^TQC\Phi_e(\tau, t) \, d\tau$$

and

$$s(t) = e^{-\theta(t-t_0)}\Phi_s^T(t, t_0)s_0 + \int_{t_0}^{t} e^{-\theta(t-\tau)}\Phi_s^T(\tau, t)C^TQs(\tau) \, d\tau$$

Writing in differential form, and after some computation, we have

$$\dot{P}(t) = -\theta - (A^T(v(t))P(t) + P(t)A(v(t)) + C^TQC$$

$$\dot{s}(t) = -\theta s(t) - A^T(v(t))s(t) + C^TQy(t)$$

where $P(t_0) = P_0$ and $s(t_0) = s_0z_0$. Then, defining $z(t) = P^{-1}(t)s(t)$ and taking the time derivative, we finally obtain the observer for state affine system

$$\dot{z}(t) = A(v(t))z(t) + P^{-1}(t)C(y - Cz),$$

$$P'(t) = -\theta P(t) - A^T(v(t))P(t)$$

where

$$P_j = \begin{pmatrix} P_{j1} & P_{j2} \\ P_{j1} & P_{j3} \end{pmatrix}$$

Developing (8) gives the following differential equations

$$\dot{P}_{j1} = \left(2\frac{r_c}{l_c} - \theta\right)P_{j1} - 2\frac{1}{C}S_jP_{j2} + 1,$$

$$\dot{P}_{j2} = \frac{1}{l_c}S_jP_{j1} + \left(\frac{r_c}{l_c} - \theta\right)P_{j2} - \frac{1}{C}S_jP_{j3},$$

$$\dot{P}_{j3} = \frac{2}{l_c}S_jP_{j2} - \theta P_{j3}$$

Besides, the correction terms are found from

$$P_j^{-1}C_j^T(y - \hat{y}) = -\frac{1}{\det P_j} \begin{pmatrix} P_{j1} \\ -P_{j2} \end{pmatrix}(i - \hat{i})$$

Thus, from (22) the correction terms for the current and capacitor voltage observers, $\xi_j$, are found as

$$\xi_{j1} = \frac{P_{j1}}{\det P_j}(i - \hat{i}), \quad \xi_{j2} = -\frac{P_{j2}}{\det P_j}(i - \hat{i})$$

where $\det P_j$ is the matrix gain determinant defined as $(P_{j1}P_{j3} - P_{j2}^2)$. Notice that the output variable is the output current, $i(t)$ (denoted only by $i$ for simplicity), being the same for all the subsystems.

The observer implementation means solving (19), and (21)–(23) for each subsystem in real time. The observer has been implemented using Simulink.

The real-time implementation of the proposed observer (block 7, Fig. 5) can be carried out by using a DSP. First, the needed signals (supply voltage, inverter phase current and inverter switching function) are acquired by the DSP at a constant sampling rate. Then, the set of differential equations cited above are to be solved by the digital processor at each sampling instant. Thus, the used DSP must be fast enough to solve all the needed equations in real time. Recently, a lot of microprocessors are programmed under a Windows graphical environment which simplifies the implementation task.

9 Appendix B: Observer implementation

The general structure of the proposed observer is given by (8), which is expressed in extended form in (11). As shown, the observer has an structure similar to that of the original interconnected subsystems, $\hat{x}_j$, plus a correction term, $P_j^{-1}C_j^T(y - \hat{y})$, which dynamic is defined by $P_j$. Then, the observer implementation means solving in real time a pair of equations, $\dot{Z}_j$ and $P_j$, for each subsystem. Let the following equation, $\dot{Z}_j$, represent the observer for subsystem $j$

$$\begin{cases}
\frac{d}{dt}\hat{x}_j(t) = \frac{r_c}{l_c}\hat{x}_j(t) - \frac{1}{l_c}S_j\hat{v}_{cj} - \frac{1}{l_c} \\
\times(S_j\hat{v}_{cj} + \cdots + S_{j-1}\hat{v}_{cj} + S_j\hat{v}_{cj+1}) + \cdots + S_n\hat{v}_{cj} + S_{n+1}\hat{v}_{cj+1} - v_{net} - \underbrace{-\xi_{j1}} \\
\frac{d}{dt}\hat{v}_{cj} = \frac{1}{C}\dot{s}_j(t) - \xi_{j2}
\end{cases}$$

where $\xi_{j1}$ and $\xi_{j2}$ are the correction terms for estimating the current and capacitor voltage, respectively. The correction terms, $\xi_{j1}$ and $\xi_{j2}$, are determined by solving the equation that describes the observer gain, $P_j$, given by (8), with

$$P_j = \begin{pmatrix} P_{j1} & P_{j2} \\ P_{j1} & P_{j3} \end{pmatrix}$$

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